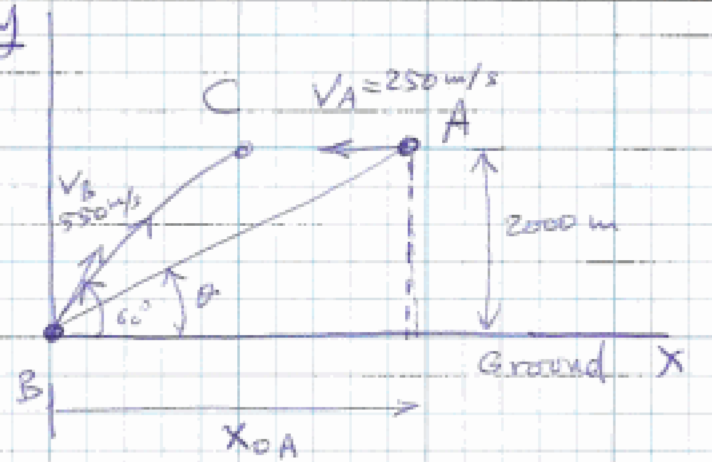
MECH 230 DYNAMICS
PREVIOUS QUIZ 1 SOLUTION

6 – 11 – 2010

No. 1



particle A

$V_A = 250$ m/s ← = constant

1. $\overset{+}{\leftarrow}$ $x_A = x_{A_0} + V_{A_0} t$

$\overset{+}{\leftarrow}$ $x_A = x_{A_0} - 250\,t$

particle B: $a_x = 0$

$V_{Bx} = 550 \cos 60° = \text{constant}$; $V_{Bx} = 275$ m/s

horizontal motion: $x = x_0 + V_{x_0} t$

$x_C = \overset{\text{origin}}{0} + 550 \cos 60°\, t$; $x_{@C} = 275\,t$

vertical motion: $V_{y_C} = V_{y_{0B}} + a_c t$

$V_{yC} = 550 \sin 60° + -9.81\,t$

$V_{y_C} = 476.3139 - 9.81\,t$

$y_c = y_0 + V_{y_0} t + \frac{1}{2} a_c t^2$; $y_c = \overset{\text{origin}}{0} + 550 \sin 60°\, t - 4.905\,t^2$

$y_c = 476.3139\,t - 4.905\,t^2$

2. @ C: $x_A = x_C$; $x_{A_0} - 250\,t = 275\,t$ ---- (1)

Find t for particle B to reach 2000 m

$2000 = 476.3139\,t - 4.905\,t^2$; $t = 4.3981$ s Ans. ← direct hit

$4.905\,t^2 - 476.3139\,t + 2000 = 0$ $t_2 = 92.7097$ s

From (1): $x_{A_0} = 250\,(4.3981) + 275\,(4.3981)$

$X_{A_0} = 2309.$ m

3. $\tan\theta = \dfrac{2000}{2309}$; $\theta = \tan^{-1}\left(\dfrac{2000}{2309}\right) = 40.9°$ Ans.

PROBLEM 2 : Given $\vec{r} = 12t^2\,\hat{i} + 3t^3\,\hat{j}$

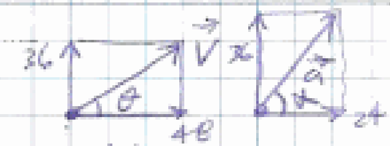
1. @ $t = 2$ Find $\vec{V}$, $\vec{a}$

$$\vec{V} = 24t\,\hat{i} + 9t^2\,\hat{j} \ \text{m/s} \ ; \ \vec{a} = 24\,\hat{i} + 18t\,\hat{j} \ \text{m/s}^2$$

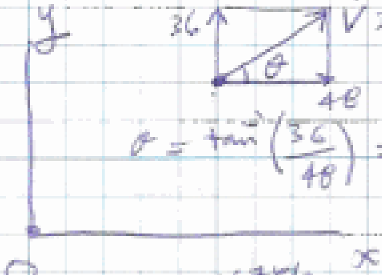
@ $t = 2$, ; $\vec{V} = 48\hat{i} + 36\hat{j}$;

$\vec{a} = 24\hat{i} + 36\hat{j}$; $\alpha = \tan^{-1}\left(\frac{36}{24}\right)$

$a = 43.267$ $\quad$ $\alpha = 56.31^\circ$



$\theta = \tan^{-1}\left(\frac{36}{48}\right) = 36.87^\circ$



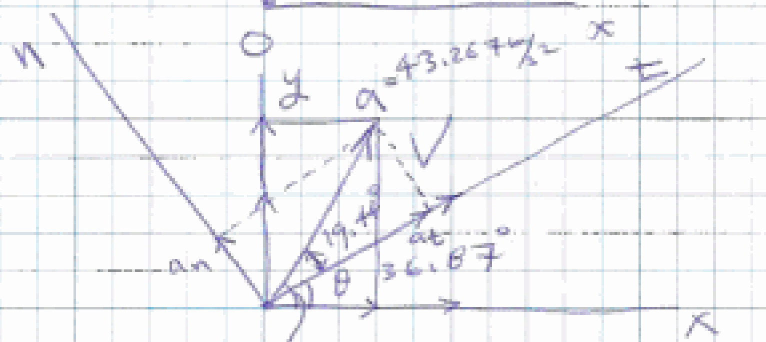
2. Find a_t, a_n @ $t = 2$s

$a_t = 43.267 \cos 19.44^\circ$

$a_t = 40.8 \ \text{m/s}^2$ Ans.

$a_n = 43.267 \sin 19.44^\circ$

$a_n = 14.4 \ \text{m/s}^2$ Ans.



$\alpha = 56.31^\circ$

$\alpha - \theta = 56.31^\circ - 36.87^\circ = 19.44^\circ$

3. Find ρ @ $t = 2$s ; @ $t = 2$s ; $V = 60$ m/s ; $V^2 = 3600$

$$a_n = \frac{V^2}{\rho} \ ; \ \rho = \frac{V^2}{a_n} = \frac{3600}{14.4} = 250 \ \text{m}$$

$\therefore \rho = 250$ m Ans.

THIS METHOD IS USED IN 3-Dimensions

Note: @ $t = 2$s, $\vec{V} = 48\hat{i} + 36\hat{j}$ m/s ; $\vec{a} = 24\hat{i} + 36\hat{j}$ m/s^2

$|\vec{V}| = (48^2 + 36^2)^{1/2} = 60$ m/s ; $|\vec{a}| = (24^2 + 36^2)^{1/2} = 43.267$ m/s^2

Angle between $\vec{a}$ & $\vec{V}$ is $\theta \Rightarrow \vec{a} \cdot \vec{V} = a.V \cos\theta$

$(24\hat{i} + 36\hat{j}) \cdot (48\hat{i} + 36\hat{j}) = 43.267 \times 60 \cos\theta$;

$1152 + 1296 = 2596.02 \cos\theta$; $\cos\theta = 0.943$; $\boxed{\theta = 19.44^\circ}$

$\therefore a_t = a\cos\theta = 43.267 \cos 19.44^\circ = 40.8$ m/s^2 Ans.

$a_n = a\sin\theta = 43.267 \sin 19.44^\circ = 14.4$ m/s^2 Ans.

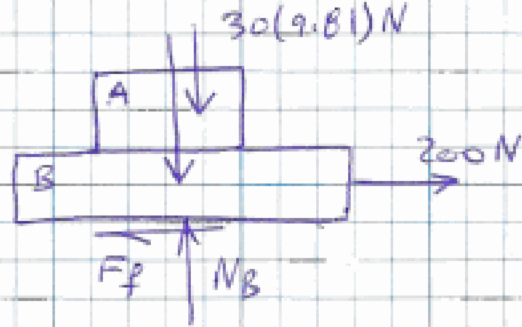
PROBLEM 3

$m_A = 20\,kg \qquad m_B = 10\,kg$

A & B $\mu_s = 0.2$, $\mu_k = 0.2$

B & Ground $\mu_s = 0.3$, $\mu_k = 0.3$

@ equilibrium:



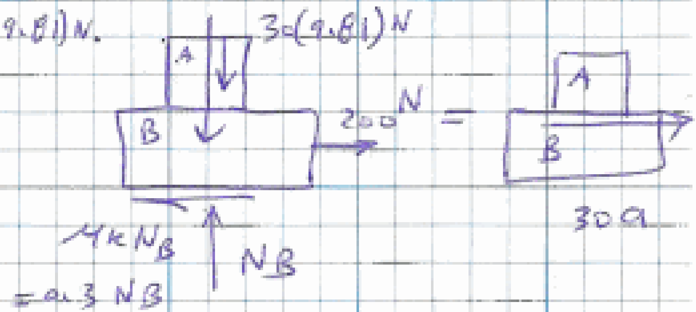
$+\uparrow \Sigma F_y = 0;\ N_B = 30(9.81)\ N$

$\xrightarrow{+} \Sigma F_x = 0; \quad 200 - F_f = 0$

$F_f = 200\ N\ ;\ F_{f_{max}} = \mu_s N_B = 0.3(30)(9.81) = 88.29\ N$

$F_{f_{present}} = 200\ N > F_{f_{max}} \Rightarrow$ slipping between B & ground.

$\therefore$ Find common acc. of A & B assuming A doesn't slip on B.



$\xrightarrow{+} \Sigma F_x = ma_x\ ;\ \Sigma F_y = 0;\ N_B = 30(9.81)\ N.$

$200 - 0.3(30)(9.81) = 30a$

$a = 3.7237\ m/s^2$

check if A slips on B?

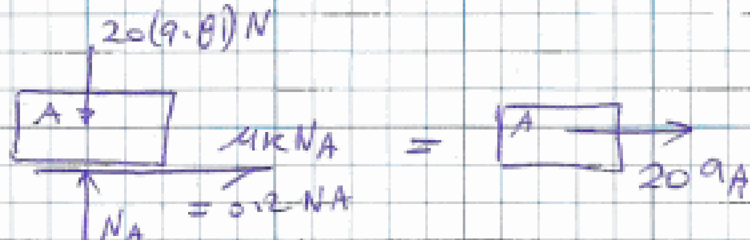
or assume A doesn't slip on B:

$+\uparrow \Sigma F_y = 0;\ N_A = 20(9.81)\ N$

$F_{f_A} = 20(3.7237) = 74.473\ N$

$(F_{f_A})_{max} = 0.2(20)(9.81) = 39.24\ N < 74.473\ N$

$\Rightarrow$ slipping between A & B.

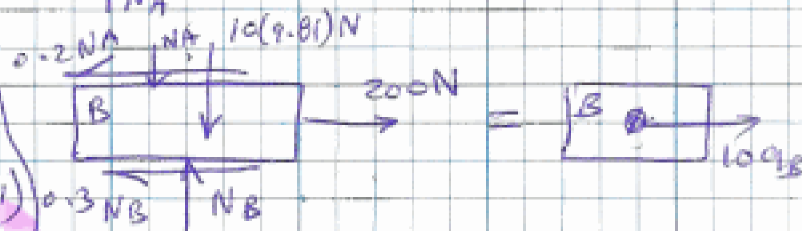


$\therefore$

$\xrightarrow{+} \Sigma F_x = ma_x$

$0.2(\cancel{20})(9.81) = \cancel{20}\, a_A$

$\boxed{a_A = 1.962\ m/s^2 \text{ Ans.}}$



$N_B = 30(9.81)\ N$

$200 - 0.2(\overset{N_A}{20})(9.81) - 0.3(30)(9.81)$

$= 10\,a_B\ ;\ a_B = 7.247\ m/s^2$

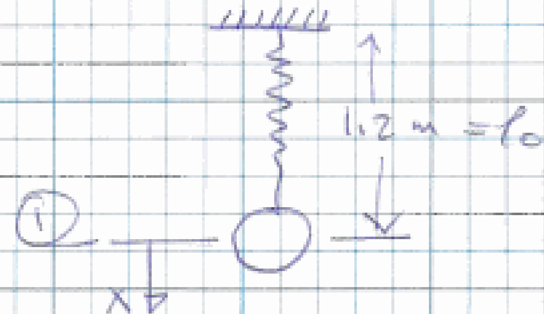
PROBLEM 4

Given:

$m = 27\ kg \qquad l_0 = 1.2\ m$

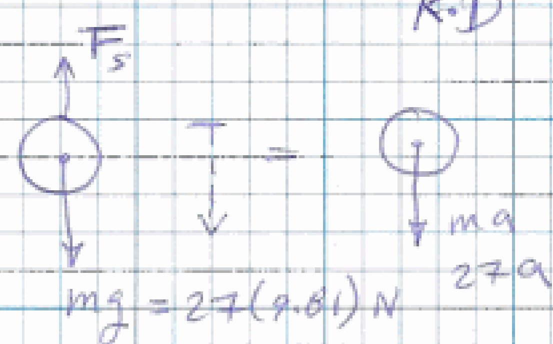
$K = 1750\ N/m$

@ $x = 0$ rest



1.

F.B.D of sphere



2. max elongation $\Rightarrow V_2 = 0$ (stop)

(vel., displ., force) $\Rightarrow$ use

$T_1 + \Sigma U_{1-2} = T_2$

$T_1 = 0^{\text{rest}}, \; T_2 = 0^{\text{stop}}, \quad \Sigma U_{1-2} = 27(9.81)\Delta X$

$\Sigma U_{1-2} = 27(9.81)\Delta X - \frac{1750}{2}\Delta X^2 + \frac{1}{2}K\cancel{S_1^2} - \frac{1}{2}KS_2^2$

$\therefore \quad \Delta X\left(27(9.81) - \frac{1750}{2}\Delta X\right) = 0$

$\Delta X = 0 \; ; \quad 264.87 - 875\,\Delta X = 0; \; \Delta X =$

$\underline{\Delta X = 0.30271\ m}$ Ans.

3. max $V. \Rightarrow a = 0 \Rightarrow \quad F_s = mg\; ; \; K\Delta X = 27(9.81)$

$\Delta X = 0.151354\ m \qquad \therefore \Delta X = \frac{27(9.81)}{1750}$

$T_1 + \Sigma U_{1-2} = T_2 \; , \quad T_1 = 0$ rest ; $T_2 = \frac{1}{2} m V_{max}^2$

$T_2 = 13.5\ V_{max}^2$

$\Sigma U_{1-2} = 27(9.81)(0.151354) - \frac{1750}{2}(0.151354)^2$

$= 40.0891 - 20.0445 = 20.0446\ J$

$\therefore \quad 20.0446 = 13.5\ V_{max}^2 \quad ; \; V_{max}^2 = 1.48478$

$\underline{V_{max} = 1.2185\ m/s}$ Ans.